Estimation of Primaries by Curvelet-domain Matched Filtering and Sparse Inversion

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Outline

• Curvelet domain matched filtering
• EPSI & Matching Surface Reflectivity
• Results
• Future Works
Successful Matching

(i) control possible overfitting

(ii) handle data with non-unique dips

(iii) apply wavefield separation after matching stably

Herrmann et al (2007)
The pseudodifferential operator

\[(\Psi f)(x) = \int_{\mathbb{R}^d} e^{-ix \cdot \zeta} a(x, \zeta) \hat{f}(\zeta) d\zeta\]

The symbol of the pseudodifferential operator
(\Psi f)(x) \approx C^T D_{\Psi} C f(x)
Curvelet Matching Formulation

\[ g = \Psi f \]

\[ z = \arg\min_z \frac{1}{2} \| g - Bz \|_2^2 \]

\[ B := C^T \text{diag}(Cf) \]
Curvelet Matching Formulation

\[ g = \Psi f \]

\[ z = \arg\min_z \frac{1}{2} \| g - Bz \|_2^2 + \frac{\lambda^2}{2} \| Lz \|_2^2 \]

\[ B := C^T \text{diag}(Cf) \]

\[ L = \begin{bmatrix} D_1^2 & D_2^T & D_\theta^T & D_{\text{scale}}^T \end{bmatrix}^T \]
Estimation of primaries by sparse inversion

\[ \hat{\mathbf{G}} \approx \mathbf{Q} + \mathbf{R}\hat{\mathbf{P}} \]

- **\hat{\mathbf{G}}**
  - Surface-free data
- **\mathbf{Q} + \mathbf{R}\hat{\mathbf{P}}**
  - Downgoing wavefield
  - Unknown source function
  - Unknown reflection operator
- **\hat{\mathbf{P}}**
  - Upgoing wavefield
EPSI (1D Case)
EPSI (1D Case)

\[ Q^+ + G - P^- \]

Source
Receivers (geophones)

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EPSI (1D Case)

Source

Q^+

G

P^-

Receivers (geophones)

P_0^-

GRP^-

Q^+
EPSI (1D Case)
EPSI (1D Case)
EPSI (1D Case)

\[ Q^+ + G - P^- - R = -1 \]

Source
 Receivers (geophones)

\[ Q^+ \]

\[ P_0^- \]

\[ GRP^- \]
EPSI (1D Case)
EPSI (1D Case)

\[ Q^+ + G - R = -1 \]

Source

Receivers (geophones)

\[ Q^+ \]
**EPSI (1D Case)**

\[ Q^+ + G - P^- = -1 \]

Source

Receivers (geophones)

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Estimation of primaries by sparse inversion

Solution via tri-convex optimization

Fix the source $Q$, assume $R = -I$ for now. Solve for the Green’s function $G$

$$\hat{x} = \arg \min_x \|x\|_1 \quad \text{subject to} \quad \|A[\hat{Q}]x - b\|_2 \leq \sigma$$

$$\hat{g} := \operatorname{vec}(\hat{G}_{1\ldots n_F}) = F^t S^* \hat{x}$$
Estimation of primaries by sparse inversion

Fix the Green’s function $G$, solve for the source $Q$.

$$\hat{q} = \arg \min_q \frac{1}{2} \| \tilde{y} - B[\hat{G}]\hat{q} \|_2^2 + \lambda_F \| L_F \hat{q} \|_2^2$$

$$B[\hat{G}] := \text{blockdiag}([\hat{G} I]_{1 \cdots n_f})$$

$$\tilde{y} = \text{vec}([\hat{P} - \tilde{G}\hat{P}]_{1 \cdots n_F})$$
Curvelet-domain matching

When $\mathbf{R} \neq -\mathbf{I}$, we can use curvelet-domain matching, i.e.,

$$\hat{\mathbf{P}} - \hat{\mathbf{G}}\hat{\mathbf{Q}} = \hat{\mathbf{G}}\hat{\mathbf{R}}\hat{\mathbf{P}}$$

with,

$$\mathbf{R} \approx \mathbf{C}^* \text{diag}(\mathbf{z})\mathbf{C}$$

to get:

$$\hat{\mathbf{P}} - \hat{\mathbf{G}}\hat{\mathbf{Q}} = \hat{\mathbf{G}}[\mathbf{C}^* \text{diag}(\mathbf{z})\mathbf{C}\hat{\mathbf{P}}]$$
\[ \hat{b} = \hat{G} \left[ C^* \text{diag}(\hat{z}) C \hat{P} \right] \]
\( \hat{b} = \hat{G} \left[ C^* \text{diag}(\hat{z}) C P \right] \)
No Matching
With Matching
Future Works

• Faster formulation

\[ P - GQ = C^* \, \text{diag}(z) \, CGP \]

• Frequency regularization

• Bayesian separation.
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