Randomized sampling “without repetition” in time-lapse seismic surveys

Felix Oghenekohwo
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Randomized undersampling
– examples from industry (ConocoPhilips)

Deliberate & natural randomness in acquisition
(thanks to Chuck Mosher)

Compressive Sensing = Acquisition Efficiency

**Time-lapse seismic**

- **Current acquisition paradigm:**
  - repeat **expensive dense** acquisitions & “**independent**” processing
  - compute **differences** between **baseline & monitor** survey(s)
  - hampered by **practical** challenges to ensure** repetition**
Time-lapse seismic

- **Current acquisition paradigm:**
  - repeat expensive dense acquisitions & “independent” processing
  - compute differences between baseline & monitor survey(s)
  - hampered by practical challenges to ensure repetition

- **New compressive sampling paradigm:**
  - cheap subsampled acquisition, e.g. via time-jittered marine undersampling
  - may offer possibility to relax insistence on repeatability
  - exploits insights from distributed compressive sensing


Sparsity-promoting recovery

\[ \tilde{x} = \arg \min_x \|x\|_1 \quad \text{subject to} \quad Ax = b \]
Framework in 4-D

\[ A_1 x_1 = b_1 \]  
\[ A_2 x_2 = b_2 \]

should \( A_1 = A_2 \) ?

what if \( A_1 \approx A_2 \) ?

what if \( A_1 \neq A_2 \) ?

**Question**: To repeat survey design or not
Idealized synthetic time-lapse data

Baseline

Monitor

4-D signal
Structure - curvelet representation

![Graphs showing the relationship between number of coefficients and magnitude of curvelet coefficients.](image-url)
Observations

Time-lapse data has structure - significant correlations

4-D signal has structure - increased sparsity

Can we exploit the structure in the vintages and the difference simultaneously?
Distributed compressive sensing
– joint recovery model (JRM)

differences

\[ x_1 = z_0 + z_1 \]
\[ x_2 = z_0 + z_2 \]

common component

\[
\begin{bmatrix}
A_1 & A_1 & 0 \\
A_2 & 0 & A_2
\end{bmatrix}
\begin{bmatrix}
z_0 \\
z_1 \\
z_2
\end{bmatrix}
=\begin{bmatrix}
b_1 \\
b_2
\end{bmatrix}
\]
**Distributed compressive sensing**

---

**joint recovery model (JRM)**

\[ \begin{align*}
    x_1 &= z_0 + z_1 \\
    x_2 &= z_0 + z_2
\end{align*} \]

\[ x_1 = \begin{bmatrix} A_1 & A_1 & 0 \\ A_2 & 0 & A_2 \end{bmatrix} \begin{bmatrix} z_0 \\ z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \]

**Key idea:**
- use the fact that different vintages share common information
- invert for common components & differences w.r.t. the common components with sparse recovery
Sparsity-promoting recovery

Joint recovery model (JRM)

$$\tilde{z} = \text{arg min}_{z} \|z\|_1 \quad \text{subject to} \quad Az = b$$

Independent reconstruction

$$\tilde{x}_i = \text{arg min}_{x_i} \|x_i\|_1 \quad \text{subject to} \quad A_i x_i = b_i, \text{ for } i = 1, 2$$
Interpretation of the model
– w/ & w/o repetition

• In an ideal world \((A_1 = A_2)\)
  
  – JRM simplifies to recovering the difference from \((b_2 - b_1) = A_1(x_2 - x_1)\)
  
  – expect good recovery when difference is sparse
  
  – but relies on “exact” repeatability...
Interpretation of the model
– w/ & w/o repetition

• In an *ideal* world \( (A_1 = A_2) \)
  ‣ JRM *simplifies* to recovering the *difference* from \( (b_2 - b_1) = A_1(x_2 - x_1) \)
  ‣ expect *good* recovery when *difference* is *sparse*
  ‣ *but* relies on “*exact*” *repeatability*...

• In the *real* world \( (A_1 \neq A_2) \)
  ‣ no absolute *control* on *surveys*
  ‣ *calibration* errors
  ‣ noise...
Stylized Examples
Sparse baseline, monitor and time-lapse signals

- $z_0$: common component
- $z_1$, $z_2$: "difference"
- $x_1$: baseline
- $x_2$: monitor
- $x_1 - x_2$: time-lapse

Signal length $N = 50$
Stylized experiments

Conduct many CS experiments to compare

- joint vs parallel recovery of signals and the difference
- recovery with completely independent $A_1$, $A_2$
- random acquisition with different numbers of samples
Stylized experiments

Conduct many CS experiments to compare

- *joint vs parallel* recovery of signals and the difference
- recovery with *completely* independent $A_1, A_2$
- *random* acquisition with different numbers of samples

\[ b_1 = A_1 x_1 \]
\[ b_2 = A_2 x_2 \]
Stylized experiments

Conduct many CS experiments to compare

- joint vs parallel recovery of signals and the difference
- recovery with completely independent $A_1$, $A_2$
- random acquisition with different numbers of samples

Run 2000 different experiments

Compute Probability of recovery
Results: independent versus joint recovery

Recovery of vintages

Recovery of difference
Observations

Joint recovery is better than independent

Improved recovery of the vintages and the difference

Requires fewer samples
With exact repetition

\[ A_1 = A_2 \]

\[ b_1 = A_1 x_1 \quad \text{and} \quad b_2 = A_2 x_2 \]

Run 10,000 different experiments

Compute Probability of recovery

Repeat experiment as before
Results: independent versus joint recovery

Recovery of vintages

Recovery of difference
**WITH** Repetition

Recovery of vintages

Recovery of difference
WITHOUT Repetition

Recovery of vintages

Recovery of difference
Observations

- Recovery of *vintages* themselves *improves without repetition*
- Recovery of *difference* *improves with repetition* because
  - *difference* is *sparse* compared to *sparsity* of *vintages*
  - does *not* recover the *vintages* themselves
Observations

- Recovery of vintages themselves improves **without** repetition
- Recovery of difference improves **with** repetition because
  - difference is sparse compared to sparsity of vintages
  - does not recover the vintages themselves

- *Do the acquisitions really have to overlap?*
Observations

- Recovery of *vintages* themselves *improves* without repetition
- Recovery of *difference* *improves* with repetition because
  - *difference* is *sparse* compared to *sparsity* of *vintages*
  - does *not* recover the *vintages* themselves

- *Do the acquisitions really have to overlap?*

![Image](image-url)
Results: recovery and overlap dependency

Recovery of vintages

Recovery of difference
Interpretation from the stylized example

- Joint recovery model (JRM) is always superior to the independent or parallel method.

- As the degree of overlap between the sampling increases, the recovery of the signals gets worse.

- Time-lapse signal recovery benefits from some overlap.
Seismic example

Time-jittered source in marine
Method

- Velocity and density model provided by BG, taken as baseline
- High permeability zone identified at a depth of ~1300m
- Fluid substitution (gas/oil replaced with brine) simulated to derive monitor velocity model
- Wavefield simulation to generate synthetic time-lapse data
Simulated original data
– time-domain finite differences

Baseline
Monitor
4-D signal

time samples: 512
receivers: 100
sources: 100

time: 4.0 ms
receiver: 12.5 m
source: 12.5 m
Conventional vs. time-jittered sources
– undersampling ratio = 2, 2 source arrays

shorter acquisition time
geometry is not the same
Measurements – undersampled and blended

baseline

monitor
Stacked sections

Original baseline

Original 4-D signal
Stacked sections

Original 4-D signal

Original 4-D signal
Stacked sections

- 50% overlap in acquisition matrices

<table>
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<th>Time (s)</th>
<th>CMP (km)</th>
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<tr>
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<tr>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>2.5</td>
</tr>
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Parallel
(9.7 dB)

Joint
(18.2 dB)
Seismic example

An extension to model space
Example: Stacking

\[ \mathbf{b} = \mathbf{M}^H \mathbf{N}^H \mathbf{S}^H \mathbf{C}^H \mathbf{x} \]

\[ \mathbf{b} = \mathbf{Ax} \]

\[ \mathbf{m} = \mathbf{C}^H \mathbf{x} \]

- **M**: midpoint-offset
- **N**: normal move-out
- **S**: stacking
- **C**: sparsifying operator
- **H**: adjoint

observed undersampled measurements

stacked section
Idealized synthetic time-lapse data
Method

- **Acquisition**
  - Subsampled baseline and monitor data, with independent and randomly missing shots

- **Processing**
  - Independent processing of the observed data (*Parallel*)
  - Joint processing (*JRM*)
Method

• **Acquisition**
  ‣ Subsampled baseline and monitor data, with independent and randomly missing shots

• **Processing**
  ‣ Independent processing of the observed data (*Parallel*)
  ‣ Joint processing (*JRM*)

Compare *Parallel* versus *Joint*

Repeat for a “*partial*” dependence in geometry
Baseline recovery
- 0% overlap in acquisition matrices

Parallel (9.62 dB)

Joint (10.08 dB)
Monitor recovery
- 0% overlap in acquisition matrices

Parallel (10.08 dB)

residual

Joint (10.02 dB)

residual
4-D recovery
- 0% overlap in acquisition matrices

Ideal (True)

Parallel
(3.45 dB)

Joint
(7.53 dB)
Baseline recovery
- 50% overlap in acquisition matrices

Parallel
(9.62 dB)

residual

Joint
(9.79 dB)

residual
Monitor recovery
- 50% overlap in acquisition matrices

Parallel (9.69 dB) residual

Joint (9.80 dB) residual
4-D recovery
- 50% overlap in acquisition matrices

- Ideal (True)
- Parallel (7.51 dB)
- Joint (8.07 dB)
Conclusions

*Randomized* sampling techniques can be extended to time-lapse seismic surveys and processing.

Process time-lapse data *jointly*, not *independently*, in order to exploit the *shared* information.

We can work with *subsampled* data, and recover densely sampled vintages and time-lapse differences.

Provided we understand the physics of our model, we can safely work with *subsampled* data from randomized sampling ideas.

**TAKE HOME**

Think *randomized* sampling in seismic surveys!! It *saves* cost!!!
Acknowledgements

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Thank you for your attention!