A Correlation-based Misfit Criterion for Wave-equation Traveltime Tomography

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Waveform imaging
Overview

- Waveform tomography
- Wavefrontset detection
- Misfit criteria
- Numerical example
- Future work & Conclusions
Waveform tomography

Model the data as

$$\left[ \frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2 \right] u = w(t) \delta(x - s),$$

$$d(t, s, r) = u(t, x, s)|_{x=r} \equiv F[c].$$

Goal is to find the velocity given the data and source signature.
Waveform tomography

Such inverse problems have been extensively studied. Major findings:
- recovery via LS is problematic for bandlimited data
- some form of traveltime fitting needed for ‘complete’ reconstruction
Waveform tomography

Wavenumber coverage with limited aperture

[Stork; Bube; Natterer; ]
Waveform tomography

Wave-equation traveltime tomography
Waveform tomography

WE traveltime tomography:
• relies on detecting shift of singular support
• widely used criterion: maximum of the correlation

\[
\min_c \|\tau[c]\|_2^2, \quad \tau[c] = \argmax_t (d \ast \bar{d})(t)
\]

[Cara 87; Luo 91; Dahlen 10; Hormann 02; de Hoop 05; Brytik 10]
Waveform tomography
LS may be re-formulated as maximizing the normalized zero-lag correlation

\[ \| d - \bar{d} \|_2^2 = \| d \|_2^2 + \| \bar{d} \|_2^2 - 2 \left( \langle d, \bar{d} \rangle - (\bar{d}^*d)_{t=0} \right) \]

`picking approach’ is a clever extension of this
Wavefrontset detection

Given a function of the form

\[ f(x, t) = \int d\omega a(\omega, x, t) \exp[i\phi(\omega, x, t)] \]

the wavefrontset is given by

\[ \text{WF}(f) \subseteq \{ x, t; \partial_x \phi, \partial_t \phi \mid \partial_\omega \phi = 0 \} \]

In particular:

\[ \text{WF}(\bar{d} * d) \subseteq \{ s, r, \bar{T} - T; \nabla(\bar{T} - T), i\omega \} \]
Wavefrontset detection

- Multiscale WF detection via the FBI transform:

\[ G[f](t, \omega, \sigma) = \frac{1}{\sqrt{\sigma}} \int dt' f(t') W[(t - t')/\sigma] \exp[\omega t'] \]

- if \( t \not\in WF(f) \) then for fixed \( \omega \) and any \( N \in \mathbb{N} \)

\[ |G[f](t, \omega, \sigma)| \leq \sigma^N \quad \text{as} \quad \sigma \downarrow 0 \]

[Hormander 83; Hormann 02; de Hoop 05]
Wavefront set detection

reference

observed

correlation
Wavefrontset detection

reference

observed

correlation
Wavefrontset detection

reference

observed

correlation
Wavefrontset detection

reference

observed

correlation
Misfit criteria

- $\tau[\omega, \sigma] = \arg\max_t G[\bar{d} \ast d](t, \omega, \sigma)$ converges to picking approach as $\sigma \downarrow 0$ and $\omega = 0$

- **Maximize** $\| G[\bar{d} \ast d](0,.,\sigma) \|^2$

- **Minimize** $\| \partial_t G[\bar{d} \ast d](0,.,\sigma) \|^2$
Misfit criteria

Rewrite:

\[ G[f](0, \omega, \sigma) = (\hat{W}_{\sigma} \cdot f)(\omega) \]
\[ \partial_t G[f](0, \omega, \sigma) = (\hat{W}'_{\sigma} \cdot f)(\omega) \]

where

\[ W_{\sigma}(t) = \frac{1}{\sqrt{\sigma}} \exp[-(t/\sigma)^2] \]

Misfit:

\[ \phi = \frac{||W_{\sigma} \cdot (\bar{d} \ast d)||_2^2}{||d||_2^2} \]

[TvL 08; TvL 10]
Misfit criteria

velocity perturbation
Misfit criteria

velocity perturbation
Misfit criteria

velocity perturbation
Misfit criteria

velocity perturbation

small, medium, large
Misfit criteria

Multiscale WF detection allows us to move from
• Traveltime fitting at large scale to
• `Stack power’ at small scale
Numerical example II

Real cross-well data set
- Frequency domain FD
- Adjoint-state for gradient
- L-BFGS for optimization
- different stages using different basis functions

[TvL 10; ]
Numerical example
Numerical example II

`Diving-wave tomography`
Numerical example II
Reflection tomography
Reflection tomography

- Correlate wavefields in space \((\Delta x, \Delta z)\)
- Produces image volume
- Measure focusing with Gaussian weight

[Claerbout; Rickett; Sava; Symes]
Reflection tomography

Spatial correlation:

\[ E = VU^* \]

where \[ HU = Q \] and \[ H^*V = R \]

many r.h.s. !!

Action on a vector:

\[ E\mathbf{x} = V (U^*\mathbf{x}) = H^{-*} (Ry) \]

\[ y \]

one r.h.s. !!
Reflection tomography
Reflection tomography

low velocity

high velocity

[TvL 11]
Reflection tomography

c focussing power for small, medium and large scale

![Graph showing focussing power for small, medium, and large scale](image-url)
Conclusions & Future work

- Natural way to move from traveltime to amplitude fitting, and overcome loopskipping
- Multiscale WF detection might be extended to dispersion and stereo tomography
- Similar ideas might be applied in reflection case
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