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### Blind deconvolution

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**Usual assumptions and regularizations**

- $w$ is short in time
- $x$ is nearly sparse
- $\ell_2$ penalty on $w$
- $\ell_1$ penalty on $x$

### Blind deconvolution with multiples

$$ y = w \ast x = w_1 \ast x_1 + \cdots = w_n \ast x_n + x' + \cdots $$

**Traditional solver (EPSI)**

- Assume $w$ is short in time: $w \in C^0, C = \mathbb{J}, \mathbb{R}$
- Put $\ell_2$ penalty on $x$, $(x, \alpha) = \min_{x, \alpha} \{\|y - w \ast x - y + x'\|_2 + \alpha \|x\|_1\}$
- Optionally update $w$ and $x$.

### Resolving the scaling(shift) ambiguity is still there

$$ \hat{y} = x(\hat{w} - \hat{y}) = \hat{w}_0(\hat{w}_0 - \hat{y}) $$

where $\hat{w}_0 = aw$, $\hat{x}_0 = x_0 \hat{w}_0 / \alpha$

$$ \hat{w}_0 = \hat{w}_0 - \hat{y} $$

with $\alpha = 2$.

### Solving the optimization problem (method of multipliers)

**Original problem (non-convex, non-differentiable)**

$$ \min_{x \in \mathbb{R}^n} \{\|y - \hat{w} \ast x - y + x'\|_2 \} $$

subject to $\|y - \hat{w} \ast x - y + x'\|_2 \leq \epsilon$

**Box constraint**

$$ 0 \leq x_{i,j} \leq 1 $$

**Non-overlapping constraint**

$$ \sum_{i} x_{i,j} = 1 $$

**Lifting: Mitigate local minima**

$$ X, \hat{X} $$

**Lifted variables**

$$ x, \hat{x} $$

**Final optimization problem**

**Low rank penalty**

$$ \min_{x \in \mathbb{R}^n} \{\|y - \hat{w} \ast x - y + x'\|_2 \} $$

subject to $\|y - \hat{w} \ast x - y + x'\|_2 \leq \epsilon$

**Box constraint**

$$ 0 \leq x_{i,j} \leq 1 $$

**Non-overlapping constraint**

$$ \sum_{i} x_{i,j} = 1 $$

**Weights sums up to 1**

**Reconstruct $x, \hat{x}$ from $Z$**

$$ x_{i,j} = \sigma(x_{i,j}) $$

$$ \hat{x}_{i,j} = \sigma(\hat{x}_{i,j}) $$

**Pluto1.5 data**

Initial guess: $w = 0$, $x = \text{normalized random Gaussian vector}$

**Clean data, 30 traces**

**SNR 13dB, 40 traces**

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### References