Randomized wavefield inversion

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Motivation

- **Seismic data processing, modeling & inversion:**
  - firmly rooted in Nyquist’s sampling paradigm for (modeled) wavefields
  - too *pessimistic* for signals with *structure*
  - existence of sparsifying transforms (e.g. curvelets)

- **Major impediment:** “*curse of dimensionality*”
  - *acquisition* >> *processing & inversion* >> *modeling* *costs* are proportional to the *size* of *data* and *image* space

- **Solution strategy:**
  - *leverage new paradigm of compressive sensing* (CS)
    - identify simultaneous acquisition as CS
    - reduce acquisition, simulation, and inversion costs by *randomization* and deliberate *subsampling*
  - recovery from sample *rates* ≈ *computational cost proportional* to *transform-domain sparsity* of *data* or *model*
Today’s agenda

- Brief introduction to *compressive sensing*
  - *sparsifying* transforms
  - *randomized* = *incoherent* downsampling
  - *nonlinear* recovery by *sparsity* promotion

- *Sparsity-promoting recovery* from *randomized simultaneous measurements*
  - missing *separated* shots versus missing *simultaneous* shots
  - recovery from simultaneous data *with* and *without* primary prediction (CSed EPSI)

- *Joint sparsity-promoting recovery* from *randomized image volumes*
  - leverage *focusing*
  - *reduction* of model-space wavefields
Problem statement

Consider the following (severely) underdetermined system of linear equations

\[ \text{data (measurements/observations/simulations)} \rightarrow \begin{pmatrix} y \\ A \end{pmatrix} = \begin{pmatrix} \text{unknown} \\ x_0 \end{pmatrix} \]

Is it possible to recover \( x_0 \) accurately from \( y \)?
Perfect recovery

- **conditions**
  - $A$ obeys the **uniform uncertainty principle**
  - *randomized* $A$ $\leftrightarrow$ mutual incoherence
  - $x_0$ is **sufficiently sparse**

- **nonlinear** recovery procedure:

\[
\min_x \|x\|_1 \quad \text{s.t.} \quad Ax = y \quad \text{perfect reconstruction}
\]

- **performance**
  - S-sparse vectors recovered from roughly on the order of $S$ measurements (to within constant and log factors)

[Candès et al.’06]
[Donoho’06]
NAIVE sparsity-promoting recovery

\[ A^H y = A r \]

\[ y = A_{r'}^\dagger r' \]

\[ x_0 = A_{r'} y \]

inverse Fourier transform

detection + data-consistent amplitude recovery

Fourier transform

data-consistent amplitude recovery
Extensions

- Use CS principles to select *physically* appropriate
  - *measurement* basis $M = \text{random phase encoder}$
  - *randomized restriction* matrix $R = \text{downsampler}$
  - sparsifying transform $S$ (e.g. curvelets)
  - driven by signal type, physics, and type of acquisition (e.g. fMRI vs seismic)

- Sparse signal representation:

$$y = Ax_0$$

with

$$A = RMS^H$$

Selection is aimed at turning *aliases/coherent subsampling artifacts* into harmless *noise* ...
Recovery from *randomized* simultaneous measurements

Tim T.Y. Lin and Felix J. Herrmann, Designing simultaneous acquisitions with compressive sensing. Submitted Abstract, Amsterdam, 2009, EAG

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Relation to existing work

○ Simultaneous & continuous acquisition:
  – A new look at marine simultaneous sources by C. Beasley, ‘08
  – Simultaneous Sourcing without Compromise by R. Neelamani & C.E. Krohn, ’08.
  – Changing the mindset in seismic data acquisition by A. Berkout, ’08
  – Independent simultaneous sweeping - A method to increase the productivity of land seismic crews by D. Howe, M. Foster, T. Allen, B. Taylor, and I. Jack, ’08

○ Primary prediction through wavefield inversion:
  – Elimination of free-surface related multiples without need of the source wavelet by L. Amundsen, ‘01
  – Primary estimation by sparse inversion and its application to near offset reconstruction by G. van Groenenstijn and D. Verschuur, ’09
Two questions

• Question I: What is better? Having missing single-source or missing randomized simultaneous experiments?

• Comparison between different undersampling strategies for source experiments:
  – Deterministic missing shot positions
  – Randomized jittered shot positions
  – Randomized simultaneous shots

• Question II: What is better? First recover and then process or process directly in the compressed domain?

• Example: randomized primary prediction with EPSI
Interpolate

50% subsampled shot from regularly missing shot positions
Interpolate

SNR = 8.9 dB
50% subsampled shot from regularly missing shot positions
Interpolate

50% subsampled shot from randomized jittered shots
Interpolate

SNR = 10.9 dB
50% subsampled shot from randomized jittered shots
Simultaneous & continuous sources
Randomized simultaneous sweep signals

- Linearly ramping seismic sweep, 5 to 110 Hz
- Simultaneous source at all positions, each randomly phase encoded
50% subsampled shots from *randomized* simultaneous shots.
SNR = 16.1 dB
50% subsampled shot from *randomized simultaneous* shots
total data
Primary-prediction from randomized compressive data
Recovered **total** data from *randomized* compressive data.
Predicted primaries from recovered total data
Observations

- Incoherent *randomized* sampling crucial for creating favorable recovery conditions for *sparsity-promoting recovery* from "incomplete" data
  - depends on the choice of *downsampled randomization* RM
  - simultaneous acquisition is better for reconstruction

- Recovery greatly improves when estimating primaries
  - *deconvolved primaries* are *sparser* than *multiples*
  - *multiples* are mapped to *primaries*
  - example of *randomized wavefield inversion* with *reduced sizes*

- Push recovery down into processing flow, i.e., compressive processing & imaging

- Extend these ideas to imaging = model-space compressive sampling
Recovery from *randomized* image volumes


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Strategy

- Leverage CS towards solutions of wave simulation & imaging problems

- Subsample solution deliberately, followed by CS recovery

- Speedup if recovery costs < gain in reduced system size
  - computation
  - storage

- Examples:
  - compressed imaging by CS sampling in the model space
Relation to existing work

- **Simultaneous & continuous acquisition:**
  - Efficient Seismic Forward Modeling using Simultaneous Random Sources and Sparsity by N. Neelamani and C. Krohn and J. Krebs and M. Deffenbaugh and J. Romberg, ’08

- **Simultaneous simulations & migration:**
  - Phase encoding of shot records in prestack migration by Romero et. al., ’00.

- **Imaging:**
  - How to choose a subset of frequencies in frequency-domain finite-difference migration by Mulder & Plessix, ’04.

- **Full-waveform inversion:**
  - 3D prestack plane-wave, full-waveform inversion by Vigh and Starr, ’08

- **Wavefield extrapolation:**
  - Compressed wavefield extrapolation by T. Lin and F.J.H, ’07
  - Compressive wave computations by L. Demanet (SIA ’08 MS79 & Preprint)
Essentials of seismic inversion

Simulation:

\[ H[m]U = Q \quad \text{and} \quad H^*[m]V = \Delta R \]

discretized PDE (Helmholtz)

sources

variables (Earth)

solution (seismic wavefield)

adjoint solution (seismic wavefield)

residue (data)

Imaging:

\[ \hat{\delta I}(x_s, x_r, \omega) = (U \circ V^*) \]

\[ \delta m(x_s = x_r, t = 0) = \sum_{\omega} \omega^2 \text{diag}\{\hat{\delta I}\} \]
Essentials of seismic inversion

Simulation:

\[ \mathbf{H}[\mathbf{m}] \mathbf{U} = \mathbf{Q} \quad \text{and} \quad \mathbf{H}^*[\mathbf{m}] \mathbf{V} = \mathbf{\Delta R} \]

- High-dimensional solutions are extremely expensive to compute
- Inversion (e.g. via Gauss-Newton) requires multiple solves
- Number of blocks in \( \mathbf{H} \) and number of \( \text{rhs} \) determine simulation & acquisition costs

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\]

- Explicit matrix evaluations part of prestack migration are expensive, require lots of memory
- Improve recovery by formulating imaging as a CSed inversion problem where
  - off diagonals are penalized (impose focusing)
  - image recovered by wavefield inversion by joint sparsity promotion
Imaging by wavefield correlations

Creation of image volumes involves

$$\delta I(x_s, x_r, t) = F_t^* \sum_\omega \omega^2 (U \circ V^*)$$

with

$$(U \circ V^*) = \begin{bmatrix} \bar{U}_1 & \cdots & \bar{U}_{n_f} \\ \vdots & \ddots & \vdots \\ \bar{U}_{n_f} & \cdots & \bar{U}_{n_f} \end{bmatrix} \begin{bmatrix} V_1^T \\ \vdots \\ V_{n_f}^T \end{bmatrix}$$

and

$$U_f = [u_1 \cdots u_{n_f}] \text{ and } V_f = [v_1 \cdots v_{n_f}]$$

- Extremely large problem size
- Gradient updates do not account for the Hessian
- Recast imaging into a multi-D deconvolution problem supplemented by focussing
- Penalize off-diagonals as part of this focussing procedure
Wavefield focusing

Define linear mid-point/offset coordinate transformation

\[ \delta I'(m, h, t) = T_{(x_s,x_r)}^{\Delta h}(m,h) \delta I(x_s, x_r, t), \]

with \( m = \frac{1}{2}(x_s + x_r) \) and \( h = \frac{1}{2}(x_s - x_r) \)

Penalize **defocusing** via minimizing [Symes, ‘09]

\[ \|P_h I'(\cdot, h)\|_2 \text{ with } P_h \cdot = h \cdot \]

an **annihilator** that increasingly penalizes non-zero offsets.

**Remark:** conventional imaging principle

\[ \delta m = \delta I'(\cdot, h = 0, t = 0) \]
Wavefield inversion with focusing

Form augmented linear system

\[(U^* \circ S^*X) \approx V^*\]
\[P_hX \approx 0\]

with the sparsifying transform (curvelets/wavelets along depth-midpoint slices)

\[S \cdot := \text{vec}^{-1} \left( (\text{Id} \otimes C) T_0 \right) \text{vec}(\cdot)\cdot\]

and \(T_0\) source/receiver-midpoint offset mapping supplemented with the imaging condition for \(t=0\).

Formulation by wavefield inversion is a two-edged sword:
- Correct for amplitudes by wavefield inversion
- Reduce system size by compressive sampling ...
System-size reduction by CS

For each angular frequency, randomly subsample with CS matrix

$$\begin{bmatrix}
R_1^\sigma \otimes R_1^\rho \otimes R_1^\zeta \\
\vdots \\
R_{n_f'}^\sigma \otimes R_{n_f'}^\rho \otimes R_{n_f'}^\zeta 
\end{bmatrix}$$

random phase encoder

$$\left( F_3^* \left( e^{i\theta} \right) \right) F_3$$

$$\theta_w = \text{Uniform}(0, 2\pi)$$

with

$$n_f' \times n_\sigma' \times n_\rho' \times n_\zeta' \ll n_f \times n_s \times n_r \times n_z$$

Model-space CS subsampling along source, receiver, and depth coordinates.
Compressive wavefield inversion with focusing

Compressively sample augmented system

\[ \mathbf{R}_M \left( \mathbf{U}^* \circ \mathbf{S}_* \mathbf{X} \right) \approx \mathbf{R}_M \mathbf{V}^T \]

\[ \mathbf{P}_h \mathbf{X} \approx 0 \]

or

\[ \mathbf{A} \mathbf{X} \approx \mathbf{B} \]

Recover focused solution by mixed (1,2)-norm minimization

\[ \tilde{\mathbf{X}} = \arg \min_{\mathbf{X}} \| \mathbf{X} \|_{1,2} \quad \text{subject to} \quad \| \mathbf{A} \mathbf{X} - \mathbf{B} \|_{2,2} \leq \sigma, \]

with

\[ \| \mathbf{X} \|_{1,2} := \sum_{i \in \text{rows}(\mathbf{X})} \| \text{row}_i(\mathbf{X})^* \|_2 \]

and

\[ \| \mathbf{X} \|_{2,2} := \left( \sum_{i \in \text{rows}(\mathbf{X})} \| \text{row}_i(\mathbf{X})^* \|_2^2 \right)^{\frac{1}{2}} \]
Stylized example

background velocity model

perturbation
Stylized example

migrated CS image

plain migration

wavefield inversion

inverted CS image
Stylized example

Recovery from 64-fold subsampling...
Stylized example

correlation based

wavefield inversion
Stylized example

Correlation-based wavefield inversion
Common-image gathers are focused.
Observations

- **CS** provides a **new linear sampling paradigm** based on **randomization**
  - reduces *data* volumes and hence *acquisition*, *processing* & *inversion* costs
  - linearity allows for compressive processing & inversion

- **CS** leads to
  - “acquisition” of *smaller* data volumes that carry the **same information** or
  - to **improved inferences** from data using the **same** resources
  - **concrete implementations**

- **CS** combined with physics improved recovery by using
  - compressively-sampled multiples
  - focusing in the image space

- **Bottom line:** acquisition & processing & inversion costs are **no longer** determined by the *size* of the **discretization** but by **transform-domain sparsity** of the solution ...
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and... Thank you!